Analytical and numerical approaches in nonlocal and peridynamic structural mechanics

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Abstract:

This lecture is devoted to the development of size-dependent structural mechanics through integral models, also classified as strongly nonlocal approaches. The presentation focuses on the two main branches of strongly nonlocal elasticity: the strain-driven nonlocal approach (Rogula, 1965; Eringen, 1972; Wozniak, 1976; Maugin, 1979; Rogula, 1982; Kunin, 1982; Eringen, 2002) and the much more recent peridynamic approach, also called the relative displacement integral approach (Silling, 2000). It is shown, using one-dimensional problems (applied to bar or beam theories), in both statics and dynamics, that the two integral approaches—strain-driven nonlocal and peridynamic models—can be ill-posed for certain exponential kernels. Both problems can be regularized by combining a local and a nonlocal measure, leading to a well-posed integro-differential equation. The idea of mixing local and nonlocal elasticity, has been extensively developed by Eringen for two-phase strain-driven nonlocal models (see for instance Eringen, 1972). Combing local and peridynamic elasticity has been developed later by di Paola and his colleagues (Di Paola et al., 2009). Exact solutions for two-phase curvature-driven nonlocal beams with normalized exponential kernel have been derived by Challamel and Wang (2008) or Zhang et al. (2010). Exact solutions for two-phase peridynamic beams with exponential kernel are much more recent (Challamel and Zingales, 2025-a; Challamel and Zingales, 2025-b; Challamel and Aftabi-Sani, 2026). Interestingly, it has been recently shown that the two-phase peridynamic beam model can be reformulated as a two-phase curvature-driven beam model for specific kernels. Other nonlocal approaches applied to beam problems are presented, such as the nonlocal strain gradient beam model (Challamel and Wang, 2008), the micromorphic beam model (Zhang et al., 2010; Challamel et al., 2024), the nonlocal stress-driven beam model (Romano and Barretta, 2017), the two-phase nonlocal stress-driven beam model (Barretta et al., 2018) and the nonlocal stress gradient beam model (Challamel et al., 2025). For some exponential kernels, it is possible to merge the nonlocal strain gradient and nonlocal stress gradient beam models, thereby allowing for the reconciliation of stress- and strain-based formulations (Challamel et al., 2025). Both equivalent formulations can capture the so-called softening/stiffening small length scale effects. We conclude the presentation with a numerical study of the associated nonlocal eigenvalue differential problem, using several methods such as the finite difference method, the finite element method, the iterative variational method, and the exact finite element method. The study of nonlocal and peridynamic eigenvalue problems in 2D and 3D is also addressed.

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